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Electrochemical study of mass transfer in decaying annular swirl flow Part II: Correlation of mass transfer data

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This paper presents correlations of local mass transfer at the inner rod and the outer wall in annular decaying swirl flow generated by axial vane swirl generators. Four swirl generators with vane angles in the range 15–60° to the duct axis were used and experiments were carried out in a Reynolds number range 3300–50000 and at a Schmidt number of 1650. The results were correlated in the general form $Sh_x = 0.0204\, Re_x^{0.86}(1 + \tan\theta_i)^{0.53} Sc^{1/3}$, for the inner rod, and $Sh_x = 0.0224\, Re_x^{0.86}(1 + \tan\theta_o)^{0.55} Sc^{1/3}$, for the outer pipe. Comparison is made with heat transfer data for work with a similar entry configuration.

Nomenclature

c	coefficient
C	heat capacity ($\mathrm{J\,kg^{-1}\,K^{-1}}$)
d	diameter (m)
D_{AB}	diffusion coefficient ($\mathrm{m^2\,s^{-1}}$)
E	relative mean square error
$f(N)$	geometric parameter function
h	heat transfer coefficient ($\mathrm{W\,m^{-2}\,K^{-1}}$)
k	thermal conductivity ($\mathrm{W\,m^{-1}\,K^{-1}}$)
k_m	mass transfer coefficient ($\mathrm{m\,s^{-1}}$)
L	axial length (m)
n	numerical exponent
N	number of experimental data points, number of vanes
r	radial distance (m)
R	annulus diameter ratio
t	tape twist ratio in Equation 1
T	temperature (K)
U	mean axial velocity component ($\mathrm{m\,s^{-1}}$)
W	width of vanes (m)
x	axial distance (m)

Dimensionless group

Nu	Nusselt number/(hd/k), (hx/k)
Pr	Prandtl number/$(C\mu/k)$
Re	Reynolds number/(Ud/ν), (Ux/ν), (Ud_e/ν)
Sc	Schmidt number/(ν/D_{AB})
Sh	Sherwood number/$(k_m d/D)$, $(k_m x/D)$

Greek letters

θ	swirler blade angle
μ	dynamic viscosity ($\mathrm{kg\,m^{-1}\,s^{-1}}$)
ν	kinematic viscosity ($\mathrm{m^2\,s^{-1}}$)
ρ	density ($\mathrm{kg\,m^{-3}}$)

Subscripts and superscripts

a	fully developed axial value
B	bulk
cal	calculated
e	equivalent
exp	experimental
i	inner rod
L	based on length
m	mean value
o	outer wall
cal	calculated value
r	radial component
s	swirl flow
ti	tangential inlet
W	wall
x	axial component, based on length
θ	based on peak tangential velocity

1. Introduction

Use of swirl flow is a possible technique for enhancing convective transfer coefficients and has received considerable attention in heat transfer studies. Heat transfer in swirling pipe flow was investigated by Blackwelder and Kreith [1] and Klepper [2] who employed twisted tapes to obtain both developed and decaying swirl flow. Klepper correlated experimental results in the decay region by the equation

$$Nu_x = 0.023\, Re^{0.8} Pr^{0.4} \left(\frac{T_W}{T_B}\right)^{-0.5} \phi_1 \phi_2 \qquad (1)$$

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where

$$\phi_1 = \left(\frac{0.7 + 4.2 \times 10^{-5} Re}{1 + 3.9 \times 10^{-5} Re}\right)$$

$$\phi_2 = \left(1 + \frac{1.05}{[t + 2.5 \times 10^{-3}(x/d)^2]^{0.6}}\right)$$

and where t is the tape twist ratio (ratio of the tube length per 180° tape twist to the tube diameter). Blum and Oliver [3] investigated heat transfer in decaying swirl flow generated by tangential inlets. These authors suggested the following correlation for the local heat transfer coefficients for the heating of air in a pipe

$$h = 1.36 \times 10^{-5} Re_\theta^{1.38} \quad (2)$$

where Re_θ is the Reynolds number based on peak tangential velocity at each axial measurement position. Koval'nogov and Shchukin [4] developed the following correlation for water heated electrically in a pipe

$$Nu = 0.021 Re^{0.8} Pr^{0.43} \left(\frac{Pr_B}{Pr_W}\right)^{0.25} \left(1 + 0.147 \times \frac{\theta}{15}\right)^{0.82} \quad (3)$$

where θ is the swirler blade angle (degrees) near the surface of the wall. Sudarev [5] investigated heat transfer in decaying annular swirl flow, using axial guide-vane swirlers. Results for the heat transfer coefficients, averaged over a length L of test section, for electric heating of air were correlated, for $L/d \leqslant 25$, by

$$Nu_L = 0.0319\, Re_L^{0.8} (1 + \tan\theta)^{0.77} \quad (4)$$

Zaherzadeh and Jagadish [6] studied heat transfer in decaying swirl flow created by tangential vanes with water as the working fluid. The average experimental data were correlated by

$$Nu = cRe^n Pr^{0.4} \quad (5)$$

where $c, n = f(D, W, N)$, and D is the diameter of the swirler disc, W the width of the vanes and N the number of the vanes. Hay and West [7] measured local heat transfer coefficients for air flowing through a cylindrical pipe in swirling flow generated by single rectangular tangential slot and proposed the correlation

$$Nu_x = 0.0119\, Re_x^{0.8} \quad (6)$$

Only a few studies have been reported in the literature on mass transfer in decaying swirl flow. Shoukry and Shemilt [8] investigated mass transfer enhancement in decaying annular pipe flow generated by a single tangential inlet, using the electrochemical limiting diffusion current technique to measure average mass transfer coefficients. They correlated the data in the standard form

$$Sh = cRe^n Sc^{1/3} \quad (7)$$

where both c and n are functions of tangential inlet diameter and the distance of the inner rod electrode from the inlet. De Sa *et al.* [9] extended the above study to the investigation of mass transfer in the entrance region of decaying annular swirl flow using inner rod cathodes at different distances from the tangential entry. These authors found that the mass transfer is higher than that in axial flow, for $Re > 6000$, and smaller for $Re < 6000$. Legentilhomme and Legrand [10, 11], carried out very similar studies, including the effect of varying annulus ratio. It was found that the effect of initial swirl intensity, which they defined as the ratio of the fluid velocity in the tangential inlet duct to that in the annular gap, is negligible and varying annulus ratio has little effect on the mass transfer process in decaying annular swirl flow. The results of Legentilhomme and Legrand were correlated by the following equation, for $Re > 2000$

$$Sh = 0.14\, Re^{0.8} Sc^{1/3} \left(\frac{L}{d_e}\right)^{-0.26} f(R)^{0.16} \quad (8)$$

where $f(R)$ is a geometric parameter function. The same authors attempted to develop a general correlation for swirl flows with tangential inlets equal to the annular gap, which they named pure swirl flow, and larger than the annular gap, which was called contraction swirl flow. For $Re > 2000$ the resulting equation was

$$\left(\frac{Sh_s}{Sh_a}\right) = 1 + 0.32 \left[\left(\frac{0.5 d_e}{d_{ti}}\right)^2 \frac{4(1+R)}{(1-R)} Re\right]^{0.18} \left(\frac{L}{d_e}\right)^{-0.31} \quad (9)$$

where Sh_s is the Sherwood number in swirl flow, Sh_a the Sherwood number in fully developed axial flow and R the annulus diameter ratio. The authors also found that, in pure swirl flow, mass transfer was higher than that in contraction swirl flow; they also observed that there was a recirculation zone just downstream of the inlet in expansion swirl flow.

2. Experimental details

The apparatus and experimental techniques used have been described in [12, 13] and are fully available in [14]. Essentially, local measurements of mass transfer were made using microelectrodes over a distance of 750 mm downstream of bladed swirl generators at both the inner and outer surfaces of an annular flow duct with diameters $d_o = 26.1$ mm and $d_i = 12$ mm. Swirl generators with blades with nominal angles of 15°, 30°, 45° and 60° to the duct axis were used. The electrochemical limiting diffusion current method was employed with 1 mm diameter nickel microcathodes set in the surface of the nickel tube or rod macrocathode. The anode was also nickel with the potassium ferri–ferro cyanide system in aqueous sodium hydroxide as supporting electrolyte. The duct Reynolds number range was 3300 to 50000.

3. Results

The detailed local axial mass transfer measurements at the inner rod and the outer wall of the annulus,

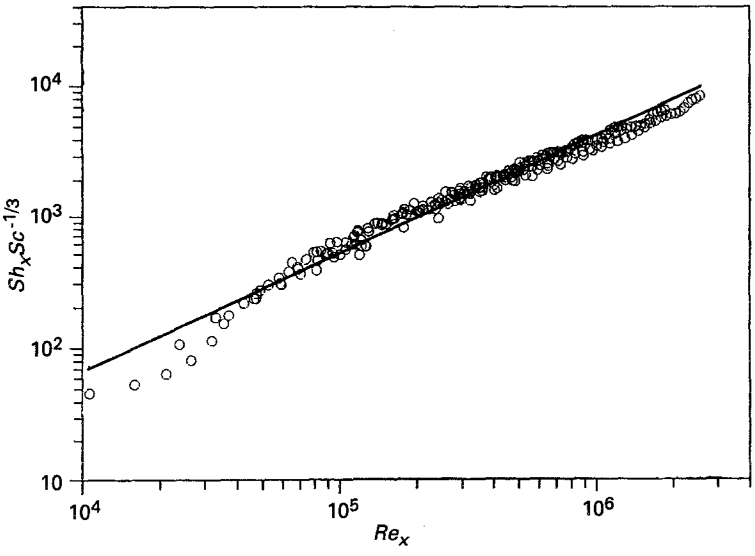

Fig. 1. Correlation of local mass transfer for the 30° swirler; inner rod. Full line: Equation 11.

including circumferential measurements and current fluctuations, have previously been presented in [12]. We now turn to approaches to data correlation for all the swirl generators for both the inner and outer annulus surfaces and over a range of Reynolds numbers.

3.1. *Correlation of experimental results for inner rod*

The experimental results were correlated by the method of least squares, using simultaneous multiple regression in the case of more than one independent variable. The individual correlations for each swirl generator for the inner rod are as follows

$$Sh_x = 0.0100\, Re_x^{0.93} Sc^{1/3} \qquad 15^\circ \text{ swirler} \qquad (10)$$

$$Sh_x = 0.0186\, Re_x^{0.89} Sc^{1/3} \qquad 30^\circ \text{ swirler} \qquad (11)$$

$$Sh_x = 0.0407\, Re_x^{0.84} Sc^{1/3} \qquad 45^\circ \text{ swirler} \qquad (12)$$

$$Sh_x = 0.1667\, Re_x^{0.74} Sc^{1/3} \qquad 60^\circ \text{ swirler} \qquad (13)$$

where $Sh_x = k_m x / D_{AB}$, and $Re_x = Ux/\nu$ and a power of one third has been fixed for the Schmidt number dependence. The correlation for the 30° swirler is shown graphically in Fig. 1.

All experimental data for the four swirl generators for the inner rod were correlated, using simultaneous multiple regression, by one general equation as a function of Reynolds number and the swirl angle, θ_i, as follows

$$Sh_x = 0.0204\, Re_x^{0.86} (1 + \tan\theta_i)^{0.53} Sc^{1/3} \qquad (14)$$

Here θ_i is the vane angle at the inner rod as given in Table 1. For the general correlation equation, the standard deviation is 6.68% and the regression coefficient is 0.986. The correlation is represented graphically in Fig. 2.

3.2. *Correlation of experimental results for outer wall*

The same procedure was applied to the experimental results for the outer wall, using the angles at the outer edge of the vanes for the general correlation equation. The correlating equations for each swirl generator are

$$Sh_x = 0.0145\, Re_x^{0.91} Sc^{1/3} \qquad 15^\circ \text{ swirler} \qquad (15)$$

$$Sh_x = 0.0263\, Re_x^{0.87} Sc^{1/3} \qquad 30^\circ \text{ swirler} \qquad (16)$$

$$Sh_x = 0.0427\, Re_x^{0.84} Sc^{1/3} \qquad 45^\circ \text{ swirler} \qquad (17)$$

$$Sh_x = 0.1042\, Re_x^{0.78} Sc^{1/3} \qquad 60^\circ \text{ swirler} \qquad (18)$$

For these equations, the standard deviations do not exceed 4.42% and the linear regression coefficients are over 0.991. The correlations for the 30° and 60° swirl generators are given in Figs 3 and 4, respectively. The following general equation, which is shown in Fig. 5, represents all the experimental data for the outer wall

$$Sh_x = 0.0224\, Re_x^{0.86} (1 + \tan\theta_o)^{0.55} Sc^{1/3} \qquad (19)$$

For this correlation, the standard deviation and the linear regression coefficient are 4.1% and 0.994, respectively.

4. Discussion

For the general correlations, the dependency of the Sherwood number on the Reynolds number is the same for the inner rod and the outer pipe (Equations 14 and 19). Even for the individual correlations, despite the small difference in the angles at the inner rod and outer edges of the vanes, the dependency of the Sherwood number on the Reynolds number can be taken to be the same for both walls of the annulus

Table 1. Geometric characteristics of the swirl generators

Designed angle/degree	*Manufactured angle* θ_i/deg.	θ_o/deg.	*Length in axial direction on outer edge* /mm
15.0	15.0	16.0	32.0
30.0	31.0	34.0	28.0
35.0	35.5	39.0	26.0
45.0	45.3	50.0	21.0
60.0	61.5	67.0	13.0

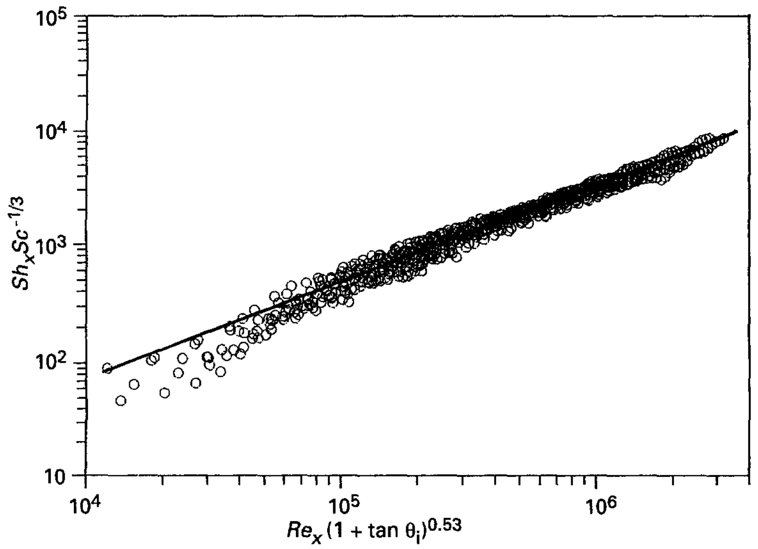

Fig. 2. General correlation of local mass transfer for the inner rod. Full line: Equation 14.

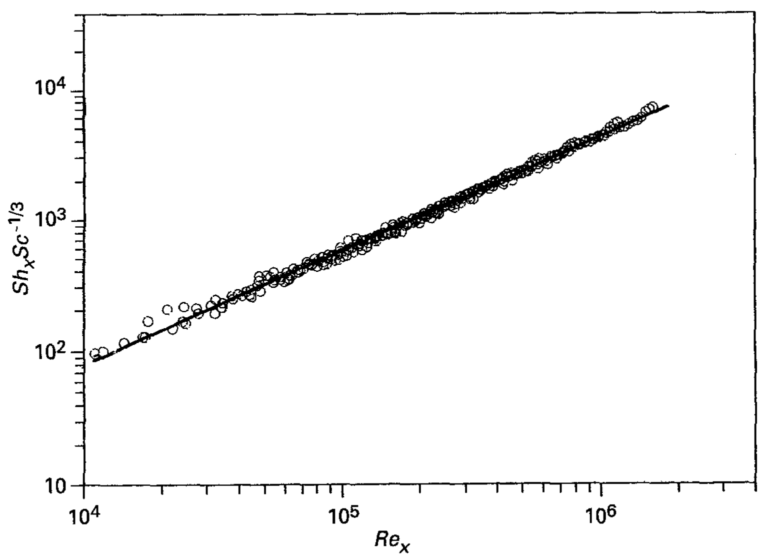

Fig. 3. Correlation of local mass transfer for the 30° swirler; outer pipe. Full line: Equation 16.

for a given swirl generator. The effect of vane angle on the Sherwood number is slightly more pronounced for the outer wall.

The relative root mean squares of the errors between the experimental values and the calculations from the correlation equations were derived from the following equation

$$E = \left[\left(\frac{1}{N} \right) \sum_{j=1}^{N} \frac{(Sh_{xj}^{\mathrm{cal}} - Sh_{xj}^{\mathrm{exp}})^2}{(Sh_{xj}^{\mathrm{cal}})^2} \right]^{1/2} \tag{20}$$

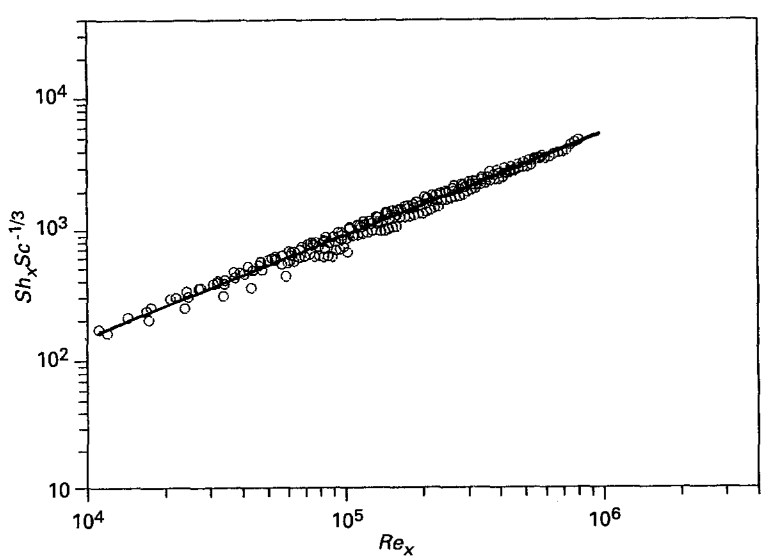

Fig. 4. Correlation of local mass transfer for the 60° swirler; outer pipe. Full line: Equation 18.

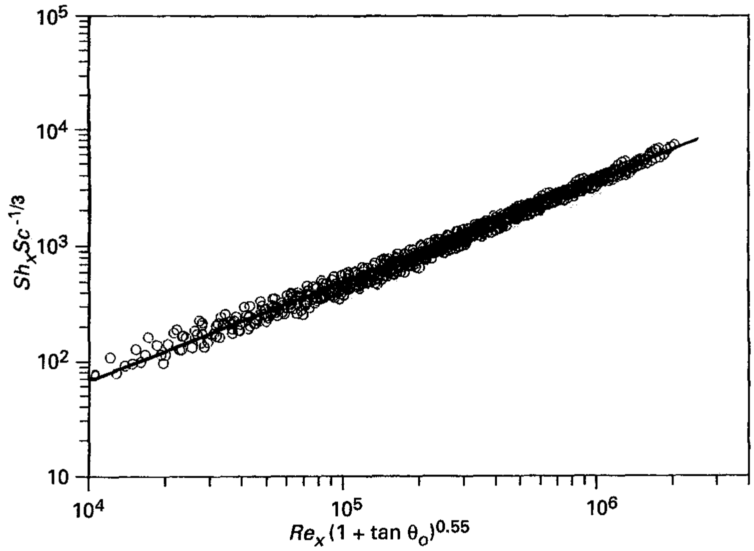

Fig. 5. General correlation of local mass transfer for the outer pipe. Full line: Equation 19.

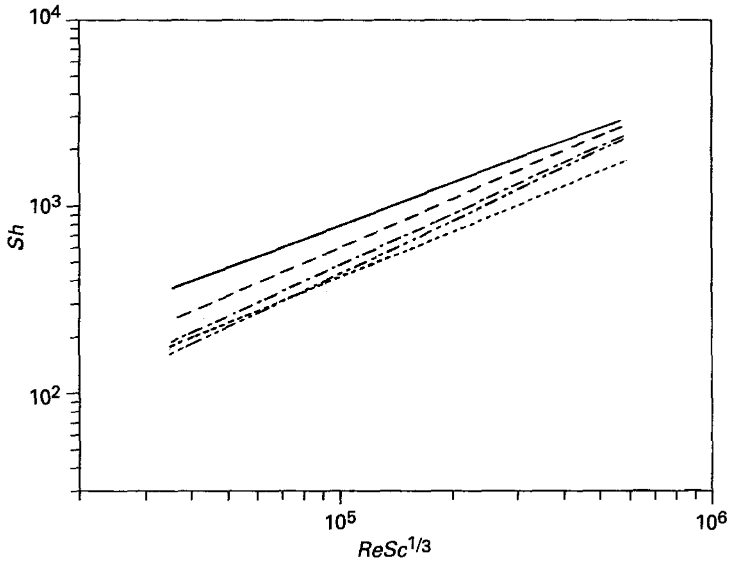

Fig. 6. Comparison of individual correlations for different swirl generators for average mass transfer behaviour for the inner rod; $L/d_e = 31$.
– – – $Sh = 0.0250\,Re^{0.80}Sc^{1/3}$ (no swirl)
– – —— $Sh = 0.0083\,Re^{0.93}Sc^{1/3}$ ($\theta_i = 15°$)
— · — $Sh = 0.0131\,Re^{0.89}Sc^{1/3}$ ($\theta_i = 31°$)
— — — $Sh = 0.0265\,Re^{0.84}Sc^{1/3}$ ($\theta_i = 45.3°$)
—— $Sh = 0.0836\,Re^{0.74}Sc^{1/3}$ ($\theta_i = 61.5°$).

where N is number of experimental data points, Sh_x^{cal} is the calculated value of the Sherwood number, and Sh_x^{exp} the experimental value of the Sherwood number. The general mass transfer correlations for the inner rod and the outer wall represent the experimental data with a relative root mean square error of 16.89% (number of experimental data points $N = 928$) and 9.37% (number of experimental data points $N = 1056$), respectively. It is the complex behaviour of the axial distribution of the local mass

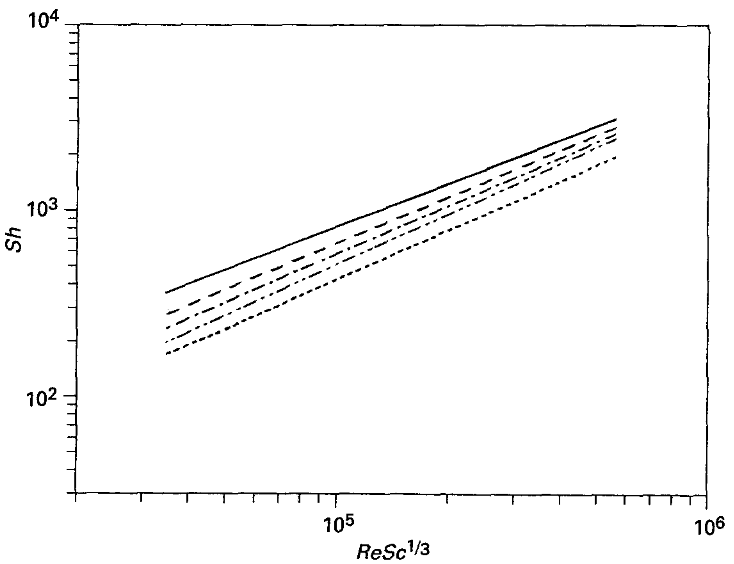

Fig. 7. Comparison of individual correlations for different swirl generators for average mass transfer behaviour for the outer pipe; $L/d_e = 31$.
- - - - $Sh = 0.0126\,Re^{0.88}Sc^{1/3}$ (no swirl)
– – —— $Sh = 0.0114\,Re^{0.91}Sc^{1/3}$ ($\theta_o = 16°$)
— · — $Sh = 0.0187\,Re^{0.87}Sc^{1/3}$ ($\theta_o = 34°$)
– – – $Sh = 0.0221\,Re^{0.84}Sc^{1/3}$ ($\theta_o = 50°$)
—— $Sh = 0.0588\,Re^{0.78}Sc^{1/3}$ ($\theta_o = 67°$)

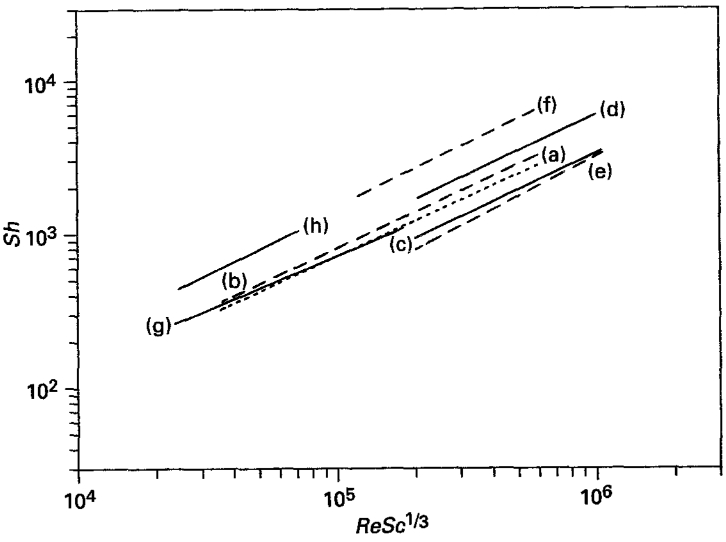

Fig. 8. Comparison of individual average correlations for different swirl generators for the inner rod and the outer pipe for the 60° swirler with correlations from the literature: (a) the inner rod, (b) the outer pipe, (c) Koval'nogov and Shchukin [4], (d) Sudarev [5], (e) Zaherzadeh and Jagadish [6], (f) Hay and West [7], (g) Shoukry and Shemilt [8], (h) Legentilhomme and Legrand [10] (see Table 2).

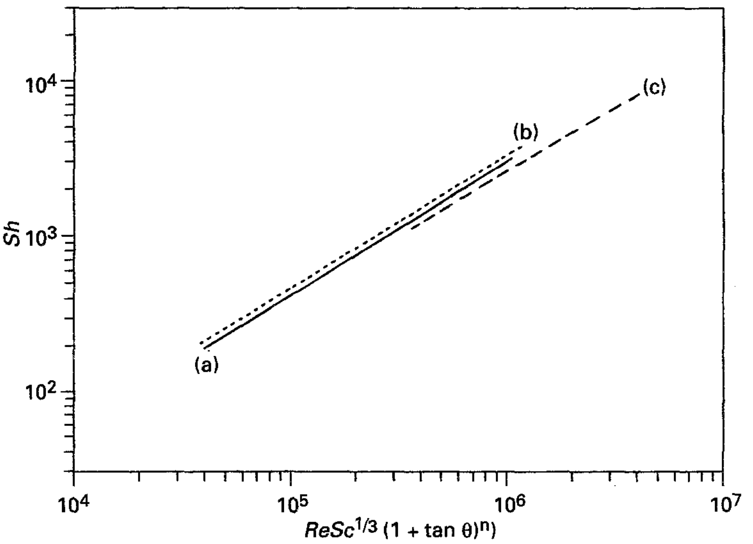

Fig. 9. Comparison of individual average correlations for different swirl generators for the inner rod and the outer pipe for the 60° swirler with the correlation of Sudarev [5]: (a) the inner rod; $n = 0.53$, (b) the outer pipe; $n = 0.55$, (c) Sudarev; $n = 0.77$ for the whole range of swirlers used, $L/d_e = 25$ (see Table 3).

transfer coefficients in the entrance region for the inner rod which contributes to this high relative root mean square error.

Similarity between different decaying swirl flow experiments is almost impossible to achieve due to the strong dependency of the flow characteristics on the specific system geometry, and the more complex flow structure and boundary conditions, compared with axial flow. But, here, comparison between the correlations of the present study and correlations of heat and mass transfer from the literature will be made, in order to highlight the effect of the geometric differences, and to ascertain whether an analogy between heat and mass transfer in decaying swirl flow is possible.

Integration over a certain test section length, $(L_2 - L_1)$, using a correlation of the form $Sh_x = cRe_x^n Sc^{1/3}$ gives the following equation

$$Sh = \frac{c}{n} Sc^{1/3} Re^n d_e^{1-n} \left[\frac{(L_2^n - L_1^n)}{(L_2 - L_1)}\right] \qquad (21)$$

where $Sh = k_m d_e / D_{AB}$ and $Re = Ud_e/\nu$.

Figures 6 and 7 give a comparison of the individual correlations for all swirl generators, averaged over a distance of $L/d_e = 31$, for the inner rod and outer pipe, respectively.

Figure 8 gives a comparison of the average individual correlations for the inner rod and the outer wall with the 60° swirl generator, and correlations for heat and mass transfer in decaying swirl flow from the literature. Details of the correlations are given in Table 2. The reason that the correlation for the 60° swirl generator is used for comparison is the fact that, in most studies, tangential inlets have been used to generate swirl. These generally give higher swirl intensities than vane swirlers and only the results for the 60° swirler are comparable to those for tangential inlets. The higher mass transfer rate results of Legentilhomme and Legrand [10] and Hay and West [7] compared with other data can be attributed to the swirl generation method. These investigators used single tangential entries to generate swirl, which results in higher swirl intensity, thus giving a high enhancement in transfer coefficient, especially immediately downstream of the swirl generator. That the predictions of Zaherzadeh and Jagadish [6] and Koval'nogov and Shchukin [4] are much lower than those of others can also be

Table 2. Correlations of average mass transfer rate in turbulent decaying swirl flow corresponding to Fig. 8

System	*Correlation equation*	*Conditions*	*Reference*
Annulus (inner rod)	$Sh = 0.0760\,Re^{0.74}Sc^{1/3}$	$3300 < Re < 50000$ $3.2 < L/d_e < 50$ vane: $\theta_i = 61.5°$	Present study
Annulus (outer pipe)	$Sh = 0.0585\,Re^{0.78}Sc^{1/3}$	$3300 < Re < 50000$ $2.4 < L/d_e < 31$ vane: $\theta_0 = 67°$	Present study
Tube (*)	$Sh = 0.0317\,Re^{0.80}Sc^{1/3}$	$1000 < Re < 90000$ $0 < L/d < 60$ vane: $\theta_0 = 67°$	Koval'nogov and Shchukin [4]
Annulus (outer pipe) (*)	$Sh = 0.0491\,Re^{0.80}Sc^{1/3}$	$3300 < Re < 97000$ $0 < L/d_e < 25$ vane: $\theta = 60°$	Sudarev [5]
Tube (*)	$Sh = 0.0111\,Re^{0.89}Sc^{1/3}$	$10000 < Re < 100000$ $0 < L/d < 100$ tangential vane	Zaherzadeh and Jagadish [6]
Tube (*)	$Sh = 0.0940Re^{0.80}Sc^{1/3}$	$10000 < Re < 500000$ $0 < L/d < 18$ tangential inlet	Hay and West [7]
Annulus (inner rod)	$Sh = 0.1170\,Re^{0.69}Sc^{1/3}$	$2000 < Re < 15000$ $0 < L/d_e < 18$ tangential inlet	Shoukry and Shermilt [8]
Annulus (inner rod)	$Sh = 0.0828\,Re^{0.80}Sc^{1/3}$	$2000 < Re < 6000$ $0 < L/d_e < 18$ tangential inlet	Legentilhomme and Legrand [10]

* By analogy from heat transfer study

attributed to the method of swirl generation used, and to the fact that they correlated the average Sherwood number for relatively long axial distances; $L/d = 60$ and 90 for these studies, respectively. Koval'nogov and Shchukin [4] designed an axial vane cascade in which the vane length in the radial direction was $r/2$. This caused a weaker swirl intensity and less heat transfer enhancement.

The plot of $\log(Sh)$ against $\log(Re(1+\tan\theta)^n Sc^{1/3})$ given in Fig. 9 shows the close agreement between the two general correlations of this study and the correlation of Sudarev [5]; details of the corresponding equations are given in Table 3. The mass transfer is higher at the outer pipe than at the inner rod in the given swirler angle range.

5. Conclusions

From the correlation of electrochemical mass transfer data in decaying annular swirl flow, in a Reynolds number range of 3300–50000 and at a Schmidt number of 1650, the dependency of the mass transfer process on the Reynolds number is found to be the same for both the inner rod and the outer pipe, but the dependency on vane angle is slightly greater for the outer pipe.

The correlations of the present study are in close agreement with those of Sudarev [5] for heat transfer, as expected from the geometric similarity of the test system; this shows that a comparison between mass and heat transfer is possible for geometrically

Table 3. Equations of average turbulent mass transfer rate correlations in Fig. 10

System	*Correlation equation*	*Conditions*	*Reference*
Annulus (inner rod)	$Sh = 0.0144\,Re^{0.86}Sc^{1/3}(1+\tan\theta_i)^{0.53}$	$3300 < Re < 50000$ $3.2 < L/d_e < 50$ vane: $15° < \theta_i < 61.5°$	Present study
Annulus (outer pipe)	$Sh = 0.0158\,Re^{0.86}Sc^{1/3}(1+\tan\theta_o)^{0.55}$	$3300 < Re < 50000$ $2.4 < L/d_e < 31$ vane: $16° < \theta_o < 67°$	Present study
Annulus (outer pipe)	$Sh = 0.0193\,Re^{0.80}Sc^{1/3}(1+\tan\theta)^{0.77}$	$17000 < Re < 97000$ $0 < L/d_e < 25$ vane: $50° < \theta < 78°$	Sudarev [5]

similar systems. It should, however, be stated that the findings of the present study are valid only for the specific geometry used.

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